

Gyroscopic Effect on the Ball of Bearing with Transient and Variation Calculus Approach for Crack Analysis on Defective Bearings.

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ABSTRACT

In this Paper an attempt is made in applying the gyroscopic principle, that the ball spinning in the bearing in any plane if dislocated in the downward direction due to any defect on the outer ring under a constant axial load, then a gyroscopic reactive couple tends to turn the ball in left direction. Thus there will be more impact of the ball with the left side of the bearing thus result in more vibration on that side. Further using the Euler's equation from calculus of variation, the mechanism and path of propagation of the crack in bearings is converted into an equation and proved experimentally. Using Finite element software transient analysis is carried out to plot the time domain graph for defect on the inner and outer race along with stress analysis. An experimental test rig is designed and fabricated along with defective bearings to test the gyroscopic and transient analysis done using a Computer (PC) based FFT (Fast Fourier transform) analyzer.

Keywords— Euler's equation, reactive couple, stress, vibrations

ARTICLE INFO

Article History

Received : 18th November 2015

Received in revised form :

19th November 2015

Accepted : 21st November , 2015

Published online :

22nd November 2015

I. INTRODUCTION

In this paper Condition monitoring of bearing is carried out by throwing light on different parameters related to the defects in the inner and outer ring of the bearing respectively. An experiment test rig is developed to study the vibration effect due to defect on the inner and outer race and comparing it with the transient analysis done in time domain in Finite element computational software. Stress analysis is also carried out to identify the stress concentration is more in the defect in inner race or outer race under dynamic conditions. For the gyroscopic effect, from the time domain analysis we can clearly distinguish between the increase in vibrations of the bearing from the change in the position of the probe attached on the either side. Basically in bearings when there is a surface contact

between the balls and the inner or outer ring for a long period of time, Due to fatigue a crack is developed and as the bearing is operated for more period of time this crack grows and moves in a particular selected path. We cannot exactly predict this path but we can derive some equations which can give an approximate path of motion for that crack. In simple words these equations can give a solution for the path of the crack which can be a straight line or it can be curve, a circle, a parabola, ellipse, hyperbola etc.

II. GYROSCOPIC ANALYSIS OF BEARING

A. Theoretical analysis of gyroscopic effect

For the gyroscopic effect to take place first is the rotating body and the other is the force or the couple trying to change the orientation of axis of rotation of a rotating body

respectively. The detailed mechanism of gyroscopic effect on the ball of bearing is explained in the figure-1, Suppose there is a defect on the outer race of the bearing at inner surface, when the ball of the bearing rotating about X-axis (spin vector) collides with the defect it gets deflected or pressed downward, i.e. the front part of the ball coming in contact with the defect starts pitching downward. Now according to the gyroscopic principle we turn the spin vector (X-axis) downward at 90 degrees we get the Active gyroscopic couple and the rotation is along the active gyroscopic couple vector (Z axis) along clockwise direction. Then the reactive gyroscopic couple according to the gyroscopic rule will be on the same axis on upper side rotating in direction opposite to the active gyroscopic couple, thus this direction of reactive couple will be in anticlockwise direction making the ball to turn to the left side.

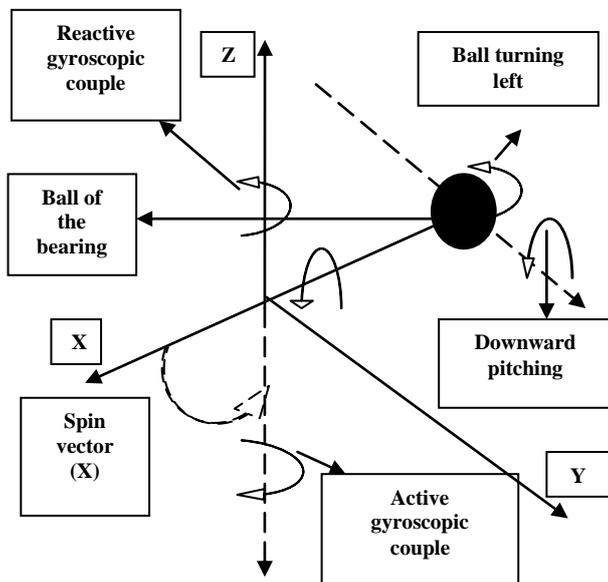


Figure-1 gyroscopic mechanism on ball of the bearing.

B. Finite element analysis of gyroscopic effect

In finite element analysis a assembly of the bearing 6202 as shown in fig-2 with its dimensions and material properties given in the below table I, IIRL is imported to the finite element analysis software, where the outer ring is fixed and rotational speed of 1200 rpm is assigned to the inner ring. Maximum load of 15N is applied on inner surface of inner ring. Stress analysis is carried out when the ball will pass towards the defect it must turn left and so the vibrations on the left side must be greater, which means that the stress on the left side must be greater than the right side. The results obtained from the finite element analysis are shown in fig-3, which shows that stress at contact between the ball and the inner race under the defect of the outer race on right side.

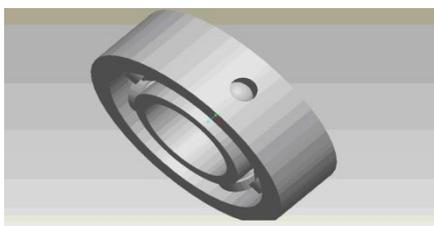


Figure-2 Outer ring defective assembly model

TABLE IV

MATERIAL PROPERTY

Density	7800 kg/m ³
Young's modulus	2.06E+11 Pascal's
Poisson's ratio	0.3

TABLE VI

DIMENSION OF BEARING

No. of balls	8
Ball diameter	6 mm
Pitch diameter	25 mm
Outer & inner ring width	11 mm
Outer bore diameter	35 mm
inner bore diameter	15 mm
Ring thickness	2 mm
Outer ring groove radius	3.24 mm
Inner ring groove radius	3.07 mm

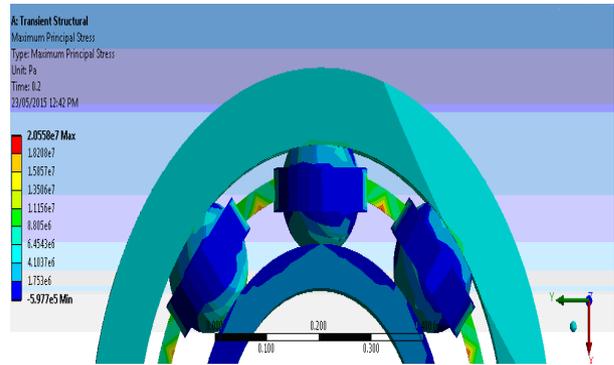


Figure-3 Stress on inner ring on the right side

Similarly in fig-4 the stress on the left hand side of the bearing is shown. From the comparison of both the result it is seen that the stress on the left hand side is greater than right hand side, as the impact of ball on left hand side is greater than that on the right hand side, which indirectly proves the gyroscopic effect.

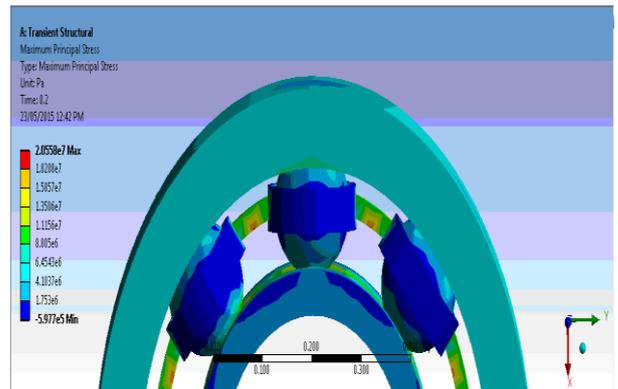


Figure-4 Stress on inner ring on the left side

C. Experimental validation of gyroscopic effect.

For the experiment validation of the gyroscopic effect of the ball bearing, an experimental test rig as shown in fig -5 is developed consisting of test bearing of 6202 model. Load of 3 Kg is attached at the disk in between the two bearings providing axial load on the bearings of about 1.5 kg or 15

Newton. On one end a defect free bearing is attached and on the other a bearing with defect on outer race is attached.



Figure-5 Bearing test rig

On the outer race a defect is made of circular hole of 0.4 cm in diameter and through out the outer ring is made as shown in fig-6.



Figure-6 Bearing with defect on outer race.

The bearing is rotated with 1100 rpm and FFT analyser probe is attached on the right and left side of the defective bearing housing. Time domain graphs are plotted which are good at condition monitoring. The time domain graph for right side of bearing is shown in fig-7 and that for left side is shown in fig-8. From the comparison of both the readings it can be seen that the vibrations on left side are greater than that on the right side respectively.

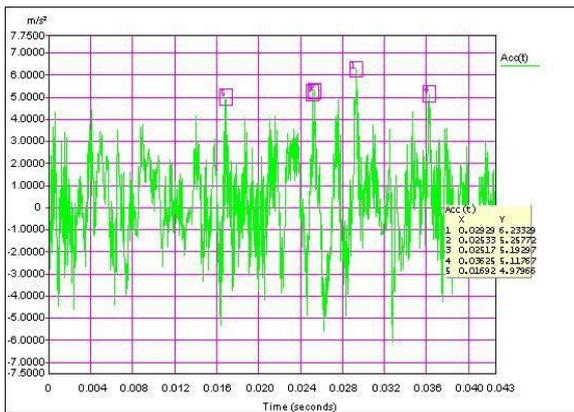


Figure-7 Time domain plot for RHS vibration

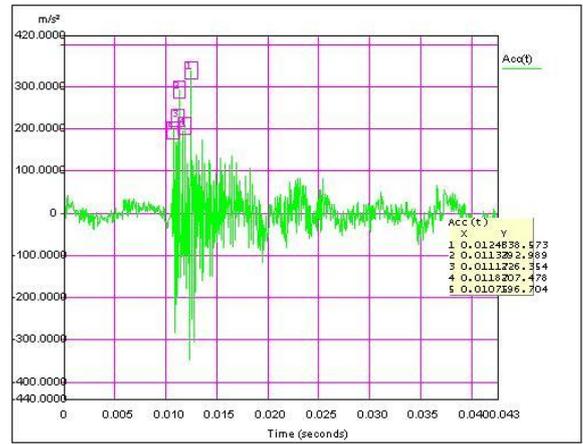


Figure-8 Time domain plot for LHS vibration

III. VARIATION CALCULUS APPROACH FOR CRACK ANALYSIS

A. Euler's equation for crack path propagation

According to the hertz contact theory for an elliptical contact between the ball and the bearing as shown in fig-10. The major and the minor axis for a quadrant is given by (a) and (b) on the x and y axis respectively. The crack is developed at the centre. The average pressure on the area is given by,

$$P_o = F/\pi ab,$$

Where F= axial load on the bearing.

The maximum pressure is given by

$$P = (3/2) \times F/\pi ab. \tag{1}$$

In the variation equation the minimum functional time (dt) is represented by.

$$\int dt = \int ds/v \tag{2}$$

We will later see the limits within which it is integrated. Here ds represent the minimum distance in a straight line from the centre to nearest point.

$$ds = \sqrt{(dx^2 + dy^2)}$$

The velocity (v) can be represented as $\rho v' n/F$

- Where, ρ = density (kg/m³)
- v' = Volume (m³)
- n = frequency or strokes per second
- F = Load on bearing (Newton)

From equation (1)

$$F = 2/3 P \pi ab$$

Substituting in equation (2) the value of F and,

$$\rho v' n/ (2/3) P \pi = A,$$

We get equation (2) in the form

$$A \int ds/ab \tag{3}$$

According to ellipse formula,

$$x^2/a^2 + y^2/b^2 = 1$$

Therefore $ab = (a^2 \sqrt{(b^2 - y^2)})/x \tag{4}$

And also, $x = a/b \sqrt{(b^2 - y^2)} \tag{5}$

The term $ds/ab =$ functional (f) and after Substituting value of ab and x from equation (4) and (5) in equation (3) we get ds/ab

$$= \sqrt{(xy^2 + 1)} x / (\sqrt{1 - (y^2/b^2)}) \times (A/ba^2)$$

Where $Xy = dx/dy$

Then by Euler's principle $df/dXy = 0$

As the functional is equal to zero it must be a constant, and shifting the term A/ba^2 on constant side we get a new constant $=C1$.

$$(Xy) x / (\sqrt{Xy^2+1}) (\sqrt{1-(y^2/b^2)}) = c/ A/ba^2 = C1$$

$$Xy = C1 (\sqrt{Xy^2+1}) (\sqrt{1-(y^2/b^2)})/x$$

Squaring both the sides,

$$(Xy)^2 = 1/(x^2-c1^2 (1-(y^2/b^2)))/C1^2 (1-(y^2/b^2))$$

Taking Xy on LHS,

$$(Xy)^2 = (1/ (x^2/ C1^2 (1-(y^2/b^2)))-1)$$

$$Xy = (1/ \sqrt{(x^2/ C1^2 (1-(y^2/b^2)))-1})$$

$$Dx/dy = (1/ \sqrt{(x^2/ C1^2 (1-(y^2/b^2)))-1})$$

$$dx = dy (1/ \sqrt{(x^2/ C1^2 (1-(y^2/b^2)))-1})$$

Solving by variable separable method,

$$dx/C1 \sqrt{1-(y^2/b^2)} = dy/ \sqrt{x^2-C1^2 (1-(y^2/b^2))}$$

$$dx/C1 \sqrt{1-(y^2/b^2)} = dy/ (\sqrt{x^2-(C1 (\sqrt{1-(y^2/b^2)}))^2}$$

Dividing numerator and denominator by x

$$dx/C1 \sqrt{1-(y^2/b^2)} x = dy/ (\sqrt{1^2-(C1/x (\sqrt{1-(y^2/b^2)}))^2} \quad (6)$$

Substituting equation (5) in equation (6)

$$dx/C1 \sqrt{1-(y^2/b^2)} x = dy/ \sqrt{1^2-(C1/a)^2}$$

Integrating on both sides

$$\text{Log } x = C1/ (\sqrt{1^2-(C1/a)^2}) [y/2b (\sqrt{1-(y^2/b^2)} + \frac{1}{2}(\text{Sin}^{-1}(y/b))]$$

Rearranging the terms,

$$\text{Sin}^{-1}(y/b) = (2\text{log}(x) (\sqrt{1^2-(C1/a)^2})/C1) - ((y/b) \sqrt{1-(y^2/b^2)})$$

Therefore, the solution is

$$y = b \text{sin} ((2\text{log}(x) (\sqrt{1^2-(C1/a)^2})/C1) + ((-y/b) \sqrt{1-(y^2/b^2)})$$

Using Euler's equation an equation is obtained

whose solution is of the form of a sin curve.

$$Y = a \text{sin} (bx+c)$$

Here a = the amplitude of vibration, comparing this equation with the Crack equation, we see that the amplitude of vibration is b , which is the length of minor axis in the elliptical area. From the analysis of the cracks it is seen that the crack when propagates from a circular point the width of the crack is equal to the diameter of the point from where it starts propagating. Therefore the crack mechanism consists of strains occurring in a diagonal manner. From the fig-10 it can be seen that when force is applied at the centre of an elliptical area, strains are developed that move in a zigzag manner like a sin curve. The area with defect experiences a pull on one side and then on the other side, and similar process continues along the vertical direction until the energy of impact gets over. The crack develops along the symmetric line of the curve.

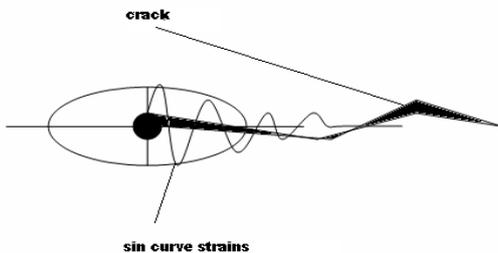


Figure-10 Crack propagation through symmetry of sin curve strains

B. Experimental validation of Euler's equation.

For the experiment a disk of metal was taken on its surface at centre a crack was made. Later on it was hammered by point or circular contact tool while the disc was rotating to develop the crack as shown in fig-11. Then

the edges of the crack were widened and if carefully observed the edges had a zigzag path. The another purpose of the experiment was to see that if the disc is like the bearing's inner and outer ring surface in contact with ball then how does the crack occurs. Using this path of crack then transient analysis is carried out in the further chapters. Also if the disc is suppose to be like a ball of the bearing spinning on X-axis as shown in fig-1 then on getting hit by the circular tool it must turn left and more impact of the tool must be on the left side of the disc. As seen from the fig-11 the crack moves in the left side instead of right side which proves the gyroscopic effect.



Fig-10 Experimental Crack formation and propagation

IV. TRANSIENT ANALYSIS FOR DEFECTIVE BEARING

A. Finite element analysis.

Transient analysis as the word suggest is an analysis carried out with respect to different intervals of time. It includes the monitoring of a particular parameter as it changes with time. In case of bearing we are going to find the time domain plot of the defect in the inner and the outer race respectively. First of all the bearing selected for the purpose of analysis is the 6202 model which finds application in the household appliance like the table fans and the ceiling fans respectively. the defective bearings assembly is made as shown in fig-11 and analysed on a FEM software under transient analysis using material property and dimension from table VI, VII, VIII.

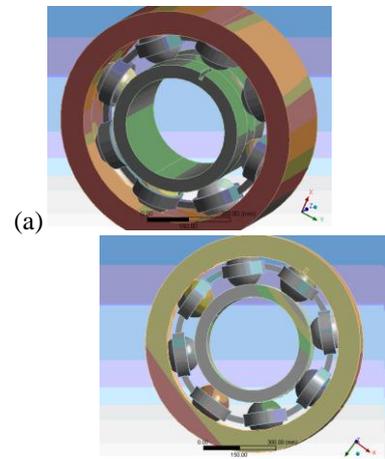


Figure-11 Bearing model with inner (a) & outer (b) race defects.

For a sample analysis, at 600 rpm & maximum load of 15N applied on the both bearings with defect on inner and outer race. With the outer ring fixed time domain graphs are plotted with stress analysis. Fig-12 shows time domain graph for inner race defect bearing and fig-13 for outer race defect bearing. Fig-14 and 15 shows the stress concentration in the bearing with inner and outer race defect.

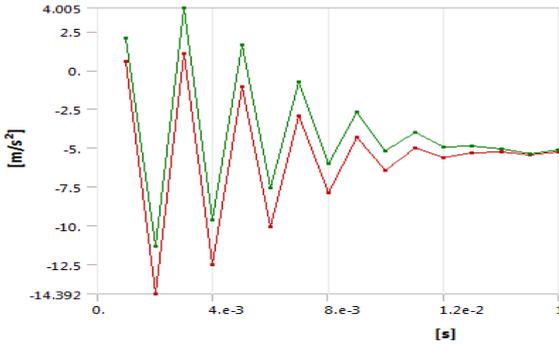


Figure-12 Time domain plot for 600 rpm for inner race defect.

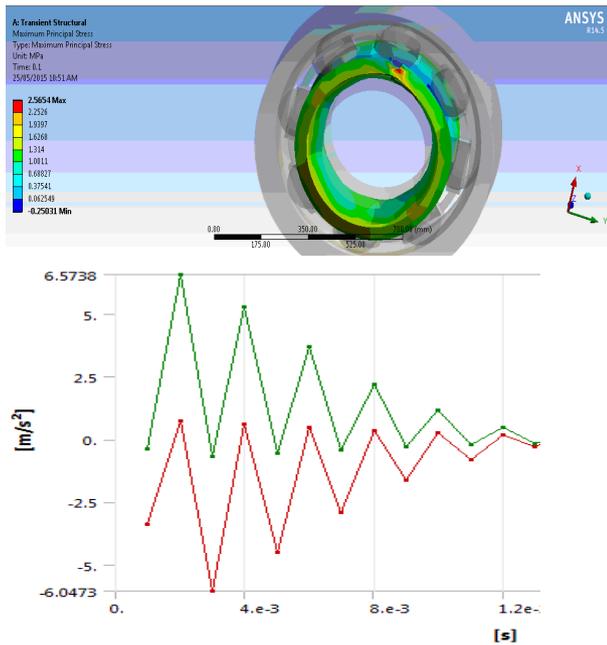


Figure-13 Time domain plot for 600 rpm for outer race defect
Fig-14 600rpm, 15 Newton inner race defect

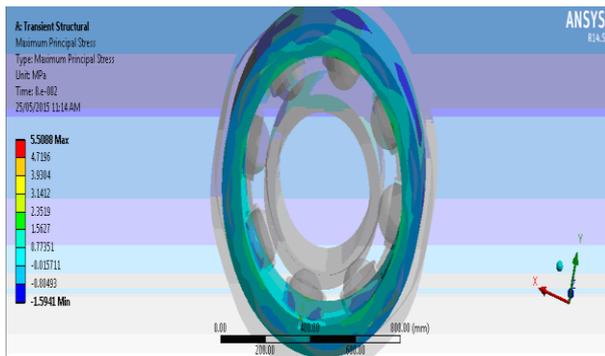


Fig-15 600rpm, 15 Newton outer race defect

From the time domain and stress analysis it is seen that the vibrations and stress concentration in bearing with outer race defect is greater than the bearing with inner race defect.

B. Experimental validation.

Using the same test rig as shown in fig-5 defect are made On outer inner race of length 3mm, width 0.5 mm and depth 1mm as shown in fig-16.. This defect is a replica of the actual cracks occurring on the bearings as verified from the Euler's equation. Experiment is carried out for 600 rpm at 15N load. The experimental set up for vibrations recording using FFT analyser is shown in fig-17. From fig-18 and 19, it can be seen that vibrations for outer race defect bearing are greater than the inner race defect bearing.



Figure-16 bearings with defect on inner and outer race



Fig-17 Experimental testing set up

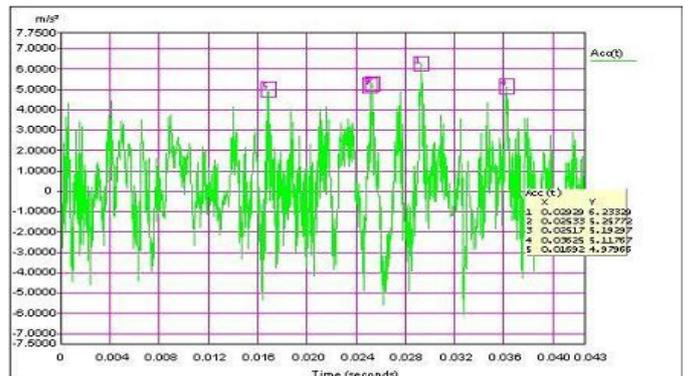


Fig-18 600rpm, 15 Newton inner race defect

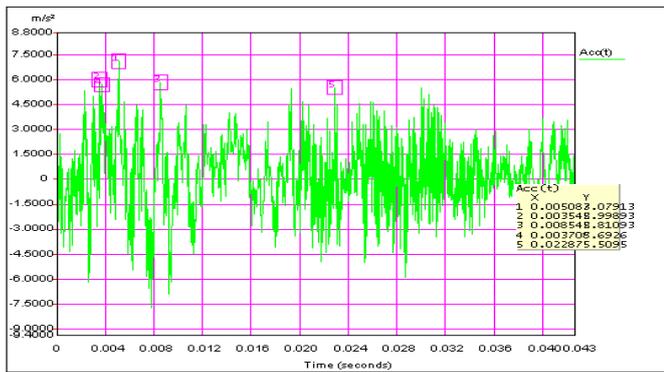


Fig-19 600rpm, 15 Newton outer race defect

If we compare the interval of time for which the transient analysis was carried out in software and the same time period in time domain we will see that the amplitude of vibration in both software and experimental in that time interval are very close which justifies the use of analysis softwares in predicting the amplitude of vibrations.

V. CONCLUSION.

From the Gyroscopic effect on the ball of the bearing it was seen that the ball of the bearing changes its direction. The theoretical, analytical and experimental results proved it. Using Euler's equation the path of propagation of crack was converted to a mathematical equation by variation calculus principle. From the experimental results gyroscopic effect was seen as well as the nature of crack was understood. Using the results from the Euler's equation similar defects was made on inner and outer race of the bearing and the intensity of vibration for which bearing is higher was found out. From the transient analysis done on the software and the experimental results it was shown that the vibrations due to the outer race defect are more than the inner race defect.

ACKNOWLEDGEMENT

This Paper is a sincere attempt to explain the gyroscopic effect on the ball of bearings using theoretical, finite element analysis and experimental method. I express my deep sense of gratitude towards Prof. Dr. Avinash Badadhe for his guidance and encouragement and the all the help he has provided me for the completion of my Paper.

REFERENCES

- [1] Arfken, Weber Harris, Mathematical methods for Physicists. 7th edition, Elsevier publications, UK 2013.
- [2] S. Timoshenko, Strength of materials, 2nd edition, D. Van Nostord Company, USA 1940.
- [3] L. Marsavina, and T. Sadowski, The influence of the interface on the fracture parameters, Damage and fracture mechanics, 1st edition, Springer publication, 2009.
- [4] Luboš Náhlík, Lucie Šestáková and Pavel Hutar, Crack Propagation in the Vicinity of the Interface Between Two Elastic Materials, 1st edition, Springer publication.2007.
- [5] M.R.M. Aliha, M.R. Ayatollahi and B. Kharazi, Numerical and Experimental Investigations of Mixed Mode Fracture in Granite Using Four-Point-

- Bend Specimen, 1st edition, Springer publication.2009.
- [6] S. Belamri, T. Tamine and A. Nemdili, Experimental and Numerical Determination of Stress Intensity Factors of Crack in Plate with a Multiple Holes, 1st edition, Springer publication.2008.
- [7] R. Tiberkak, M. Bachene, B.K. Hachi, S. Rechak and M. Haboussi, Dynamic Response of Cracked Plate Subjected to Impact Loading Using the Extended Finite Element Method (X-FEM), 1st edition, Springer publication.2008.
- [8] N Tandon, A Choudhury, "An analytical model for the prediction of the vibration response of rolling element bearings due to a localized defect", Jr. of sound and vibration, 205-3(1997),275-292.
- [9] Gunhee Jang, Seong-weon Jeong, "Vibration analysis of a rotating system due to the effect of ball Bearing waviness", Journal of sound and vibration, 269 (2004), 709-726.
- [10] N Tandon, "A comparison of some vibration measurement parameters for the condition monitoring of rolling element bearings", Elsevier journal of measurement, 12(1994)285-289.
- [11] C.K.E Nizwan, S.A. Ong, M.F.M Yusof, M.Z. Baharom, "A Wavelet decomposition analysis of vibration signal for bearing fault detection", International Conference on Mechanical Engineering Research (ICMER2013), 1-3 July 2013, Paper ID-P226 .
- [12] N Tandon, A Choudhury, "A review of vibration and acoustic measurement methods for the detection of defects in rolling element bearings", Tribology International, Volume 32, Issue 8, August 1999.
- [13] Shiyu Zhou, Jianjun Shi, "Active balancing and vibration control of rotating machinery: a survey", The Shock and vibration digest, Vol.33, No. 4, July 2001 361-371
- [14] N.Tandon, B.C. Nakra, "Comparison of vibration and acoustic measurement Techniques for the condition monitoring of rolling element bearings", Tribology International, 1992 Vol. 25, No. 3.
- [15] Taoufik Boukharouba, Mimoun Elboudjaini, Guy Pluinage. Damage and Fracture Mechanics, Failure Analysis of Engineering Materials and Structures, 1st edition, Springer publication.2009.
- [16] M Amarnath, R Shrinidhi, A Ramachandra, S B Kandagal, "Prediction of defects in antifriction bearings using vibration signal analysis", IEI journal, Vol 85, July 2004.
- [17] M.Tadina and M. Boltezar, "Improved model of a ball bearing for the simulation of vibration signals due to faults during run-up", Journal of Sound and Vibration (2011), Vol 300, issue 17.
- [18] Rajesh kumar, Divya prakash, Manpreet bains, "Identification of inner race defect in radial ball bearing using acoustic emission and wavelet analysis", proceedings of ISMA 2010.
- [19] Alireza Moazenahmadi, Dick Petersen and Carl Howard, "A nonlinear dynamic model of the vibration response of defective rolling element

- bearings”, Proceedings of Acoustics, 17-20 November 2013, Victor Harbor, Australia.
- [20] Rajesh kumar, Divya prakash, Manpreet bains, “Radial Ball Bearing Inner Race Defect Width Measurement using Analytical Wavelet Transform of Acoustic and Vibration Signal”, measurement science review, Volume 12, No. 4, 2012.
- [21] Richard G. Lyons, Understanding Digital signal processing.3rd edition, prentice hall publication, USA 2012.
- [22] A. Mohrain, “bearing defect diagnosis and acoustic emission”, Proceedings of the institution of mechanical engineers. Part J.Journal of engineering tribology, Vol 217.2012.
- [23] P.N. Bostaris, D.E koulouriotis, “A Preliminary estimation of analysis methods of vibration signals at fault diagnosis in ball bearings”, 4th international conference on NDT,Hellenic society for NDT,11-14 october 2007,Greece
- [24] A.L. Mckelvey and R.O. Ritchie, “Fatigue-Crack Growth Behavior in the Superelastic and Shape-Memory Alloy Nitinol”. Metallurgical and material transactions, Vol 32, March 2001.
- [25] Wenyi Yan, Chun Hui Wang, Xin Ping Zhang, Yiu-Wing Mai, “Theoretical modelling of the effect of plasticity on reverse transformation in superelastic shape memory alloys”. Materials Science and Engineering A354 (2003) 146_/157.